Seat No. : _______________

N13-116

November-2014

B.Sc., Sem.-V

MAT-301 : Mathematics

(Linear Algebra-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory and carry equal marks.
(2) Notations are usual everywhere.
(3) Figures to the right, indicate marks of the question/sub-question.

1. (a) If $S : U \to V$ and $T : U \to V$ are linear maps then prove that $S + T : U \to V$ is linear. **7**

OR

If $T : U \to V$ is linear map, $v_0 \in R(T)$ and if $T(u) = \overline{0}_v$ has a nontrivial solution $u \neq \overline{0}_u$ then prove that the operator equation $T(u) = v_0$ has an infinite number of solutions.

(b) If a linear map $T : V_3 \to V_2$ is defined as $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 - x_3)$, $\forall (x_1, x_2, x_3) \in V_3$ then solve the operator equation $T(x_1, x_2, x_3) = (6, 3)$. **7**

OR

Find the dual basis of the basis $B = \{(1, 0, 0), (1, 2, 0), (1, 2, 3)\}$ of the vector space V_3.

2. (a) Derive the triangle inequality by using the Cauchy-Schwarz inequality. **7**

OR

Prove that an orthonormal set of an inner product space is always linearly independent.

(b) Apply the Gram-Schmidt orthogonalization process to the basis $B = \{(1, 1, 1), (1, 2, 1), (1, 1, 3)\}$ in order to get the orthonormal basis for R^3. **7**

OR

If for $x = (x_1, x_2)$, $y = (y_1, y_2) \in V_2$ the map $\bullet$ is defined as

$$x \bullet y = \frac{1}{4}(x_1 - x_2)(y_1 - y_2) + \frac{1}{4}(x_1 + x_2)(y_1 + y_2)$$

then show that $\bullet$ is am inner product on V_2

3. (a) If $i \neq j$, $\alpha \in R$ and if $det : V^n \to R$ is a function satisfying the expected properties of the determinant then prove the followings : **7**

(i) $\det (v_1, v_2, \ldots, v_j' + v_j'', \ldots, v_n) = \det (v_1, v_2, \ldots, v_j', \ldots, v_n) + \det (v_1, v_2, \ldots, v_j'', \ldots, v_n)$.

(ii) $\det (v_1, v_2, \ldots, v_i, \ldots, v_j, \ldots, v_n) = \det (v_1, v_2, \ldots, v_i + \alpha v_j, \ldots, v_j \ldots, v_n)$

OR

State and prove the Cramer's rule of solving the system of linear equations.

(b) Use the Cramer's rule to solve :

$$x + y = 1$$
$$y + z = 3$$
$$z + x = 5$$ **7**

OR

Compute det A without expansion if $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 4 & 8 & 0 \\ 5 & 6 & 3 & 1 \end{pmatrix}$.

4. (a) Define eigen value/vector of an endomorphism. Also discuss eigen value/vector of $T : V_2 \to V_2$ defined as $T(x_1, x_2) = (x_2, x_1)$, for $(x_1, x_2) \in V_2$, if exists. **7**

OR

State and prove the Cayley-Hamilton's Theorem.

(b) If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ then find eigen values and their corresponding eigen vector of A. **7**

OR

Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, if possible.

5. Answer the following questions in short : **14**

(a) Define the space L(U, V) and a bilinear form.

(b) Define a linear functional and dual space of a vector space.

(c) State the Cauchy-Schwarz inequality.

(d) Fine det A, if $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 4 & 3 & 3 \end{bmatrix}$.

(e) Define an inner product and an inner product space.

(f) Define an orthogonal set and give one example of it.

(g) State the spectral theorem.